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BAYESIAN ESTIMATION FOR RANDOM PANEL DATA MODEL WITH APPLICATION

AMEERA JABER MOHAISEN[1] & SAJA YASEEN ABDULSAMAD[2]

[1]Assistant Professor, Department of Mathematics, College of Education for Pure Science, AL-Basrah University, Iraq

[2]Dissertation Scholar, Department of Mathematics, College of Education for Pure Science, AL-Basrah University, Iraq

ABSTRACT

In this paper, we consider the random effect panel data model which has fixed and random effects as well as the experimental error term. Bayesian approach employed to make inferences on the model coefficients. To illustrated the effectiveness of the methodology. We have chosen a data set from gross fixed capital formation and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.).The data are analysed according to our methodology by using gretl, R and matlab softwares.

KEYWORDS: Panel Data Model, Likelihood function, Bayesian approach, Markov chain Monte Carlo (MCMC), Prior distribution, Posterior distribution, Bayes factor, gross fixed capital formation, gross domestic product, economic activities for public sector, current prices

INTRODUCTION

Linear models play a central part in modern statistical methods. On the one hand, these models are able to approximate a large amount of metric data structures in their entire range of definition or at least piecewise. The theory of generalized models enables us, through appropriate link function, to apprehend error structures that deviate from the normal distribution, hence ensuring that a linear model is maintained in principle. Linear statistical methods are widely used as part of this learning process. In the biological, physical, and social sciences, as well as in business and engineering, linear models are used in both the planning stages of research and analysis of the resulting data. As well as and to the best for our knowledge the Linear models and Bayesian models were studied by many researchers for example see,[2],[3],[4],[5],[7],[8],[11],[12],[13],[14],[16].

In this paper, we consider the random effect panel data model. Our paper is related to the previous works [9], [10], which provides of theoretical results for panel data model as well as Bayes panel data model based on Markov Chain Monte Carlo (MCMC). Consider the model

$$Y_{it} = \mu + \sum_{j=1}^{K} \beta_j X_{jit} + \varepsilon_{it}, i = 1, \dots, N, t = 1, \dots, T, \quad \textbf{(1)}$$

Where, Y_{it} the value of response variable for i^{th} unit at time t, $X_{jit}$the explanatory variables, $\mu, \beta_j, j = 1, \dots, K$ are fixed parameters and ε_{it} is an error term with $\varepsilon_{it} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$.

Now, if the parameter μ is specified as:

$$\mu = \beta_0 + u_i, \quad \textbf{(2)}$$

Where, $u_i \sim N(0, \sigma_u^2)$, then, the model (1) is

$$Y_{it} = \beta_0 + \sum_{j=1}^{K} \beta_j x_{jit} + u_i + \varepsilon_{it}. \tag{3}$$

The model (3) is rewrite as follows

$$Y_{it} = \beta_0 + \sum_{j=1}^{K} \beta_j x_{jit} + \omega_{it}, \tag{4}$$

Where, $\omega_{it} = u_i + \varepsilon_{it}$, $\omega_{it} \sim N(0, \sigma_\omega^2)$, $\sigma_\omega^2 = \sigma_\varepsilon^2 + \sigma_u^2$, thus by using matrix notation the model (4) is

$$Y = F\theta + \omega \tag{5}$$

Where, $F = [e, X]$, $e = [1,1,\dots,1]^T$ has length NT $Y = [Y_{11},\dots,Y_{1T},Y_{21},\dots,\dots Y_{2T},\dots,Y_{N1},\dots,Y_{NT}]^T$ has length NT, $X = [X_1, X_2, \dots, X_N]^T$ is a $NT \times K$ design matrix of fixed effects, $\theta = [\beta_0, \beta_1, \dots, \beta_K]^T$ has length $K+1$, and $\omega = [\omega_{11},\dots,\omega_{1T},\omega_{21},\dots\omega_{2T},\dots,\omega_{N1},\dots,\omega_{NT}]^T$

Has length NT. From model (5), we have $Y \sim N(F\theta, \Psi)$, where

$\Psi = E(\omega\omega^T) = I_N \otimes (\sigma_\varepsilon^2 I_t + \sigma_u^2 ee^T) = \sigma_\varepsilon^2 (I_N \otimes I_t) + \sigma_u^2 (I_N \otimes ee^T)$,

Replace I_T by $(E_T + J_T)$ and ee^T by $T J_T$, where $J_T = \frac{1}{T} ee^T$ and $E_T = I_T - J_T$, then

$\Psi = \sigma_\varepsilon^2 [I_N \otimes (E_T + J_T)] + \sigma_u^2 (I_N \otimes T J_T)$

$= \sigma_\varepsilon^2 (I_N \otimes E_T) + \sigma_\varepsilon^2 (I_N \otimes J_T) + T\sigma_u^2 (I_N \otimes J_T)$,

By collecting terms with the same matrices, we get

$\Psi = \sigma_\varepsilon^2 (I_N \otimes E_T) + (\sigma_\varepsilon^2 + T\sigma_u^2)(I_N \otimes J_T) = \sigma_\varepsilon^2 Q + \sigma_1^2 P$, where, $\sigma_1^2 = (\sigma_\varepsilon^2 + T\sigma_u^2)$ and

$\Psi^{-1} = \frac{Q}{\sigma_\varepsilon^2} + \frac{P}{\sigma_1^2}$, $|\Psi|$ = product of its characteristic roots, [1] $\rightarrow |\Psi| = (\sigma_\varepsilon^2)^{N(T-1)} (\sigma_1^2)^N$.

The likelihood function is the joint density of the $Y's$ that is

$$L(Y; \theta, \Psi) = (2\pi)^{\frac{-NT}{2}} |\Psi|^{\frac{-1}{2}} exp\{\tfrac{-1}{2}(Y - F\theta)^T \Psi^{-1} (Y - F\theta)\}$$

$= (2\pi)^{\frac{-NT}{2}} (\sigma_\varepsilon^2)^{\frac{-N(T-1)}{2}} (\sigma_1^2)^{\frac{-N}{2}} \; exp\, \{\frac{-1}{2} (Y - F\theta)^T \left[\frac{Q}{\sigma_\varepsilon^2} + \frac{P}{\sigma_1^2}\right] (Y - F\theta)\}$.

Then, the likelihood estimators of parameters $\theta, \sigma_\varepsilon^2, \sigma_1^2$ are [9]

$\hat{\theta} = (F^T \Psi^{-1} F)^{-1} (F^T \Psi^{-1} Y)$, $\hat{\sigma}_\varepsilon^2 = \frac{1}{N(T-1)} (Y - F\hat{\theta})^T Q (Y - F\hat{\theta})$ and $\hat{\sigma}_1^2 = \frac{1}{N} (Y - F\hat{\theta})^T P ((Y - F\hat{\theta})$

The panel data model has been investigated by many researchers for example see [1],[6],[9], [10], [15]. To illustrated the effectiveness of the methodology. We have chosen a data set from gross fixed capital formation and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.).The data are analysed according to our methodology by using gretl, R and matlab softwares.

The Prior and Posterior Distributions

To specify a complete Bayesian model, we need a prior distribution on $(\theta, \sigma_\varepsilon^2, \sigma_1^2)$. We will use the uniform distribution U(0,1) of the vector parameters θ, as well as we will assume that the prior distribution on σ_ε^2 and σ_1^2 are invers gamma with of parameters α_ε, β_ε, α_1 and β_1 respectively, [10]

$\pi_0(\sigma_\varepsilon^2) = \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)}(\sigma_\varepsilon^2)^{-(\alpha_\varepsilon+1)} exp\left(-\frac{\beta_\varepsilon}{\sigma_\varepsilon^2}\right)$ and

$\pi_0(\sigma_1^2) = \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}(\sigma_1^2)^{-(\alpha_1+1)} exp\left(-\frac{\beta_1}{\sigma_1^2}\right)$, where α_ε, β_ε, α_1 and β_1 are hyperparameters that determine the priors and must be chosen by the statistician.

From the model (5) we have $Y|\theta, \sigma_\varepsilon^2, \sigma_1^2 \sim N_{NT}(F\theta, \Psi)$. Then, the likelihood function is

$L(Y|\theta, \sigma_\varepsilon^2, \sigma_1^2) = \prod_{\substack{i=1 \\ t=1}}^{NT} (2\pi)^{\frac{-1}{2}} |\Psi|^{-\frac{1}{2}} exp\{-\frac{1}{2}(Y - F\theta)^T \Psi^{-1}(Y - F\theta)\}.$

In the exponent, we add and subtract $F\hat{\theta}$ to obtain, [10]

$$[(Y - F\theta)^T \Psi^{-1}(Y - F\theta)] = [(Y - F\hat{\theta} + F\hat{\theta} - F\theta)^T \Psi^{-1}(Y - F\hat{\theta} + F\hat{\theta} - F\theta)]$$

$$= \left[(Y - F\hat{\theta}) - F(\theta - \hat{\theta})\right]^T \Psi^{-1}\left[(Y - F\hat{\theta}) - F(\theta - \hat{\theta})\right]$$

$$= [(Y - F\hat{\theta})^T \Psi^{-1}(Y - F\hat{\theta}) - (Y - F\hat{\theta})^T \Psi^{-1} F(\theta - \hat{\theta}) -$$

$(\theta - \hat{\theta})^T F^T \Psi^{-1}(Y - F\hat{\theta}) + (\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})],$

Since, $(F^T \Psi^{-1} F)\hat{\theta} = F^T \Psi^{-1} Y$, then

$$[(Y - F\theta)^T \Psi^{-1}(Y - F\theta)] = (Y - F\hat{\theta})^T \Psi^{-1}(Y - F\hat{\theta}) + (\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta}). \quad \textbf{(6)}$$

The joint posterior density of the coefficients θ and the variances σ_ε^2 andσ_1^2 givenby the expression

$\pi_1(\theta, \sigma_\varepsilon^2, \sigma_1^2|Y) \propto L(Y|\theta, \sigma_\varepsilon^2, \sigma_1^2)\pi_0(\theta, \sigma_\varepsilon^2, \sigma_1^2)$

$$\propto (2\pi)^{\frac{-NT}{2}} (\sigma_\varepsilon^2)^{\frac{-N(T-1)}{2}} (\sigma_1^2)^{\frac{-N}{2}} exp\{-\frac{1}{2}(Y - F\hat{\theta})^T \left(\frac{Q}{\sigma_\varepsilon^2} + \frac{P}{\sigma_1^2}\right)(Y - F\hat{\theta})\} exp\ \{-\frac{1}{2}(\theta - \hat{\theta})^T F^T \Psi^{-1} F$$

$$(\theta - \hat{\theta})\} \times \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)}(\sigma_\varepsilon^2)^{-(\alpha_\varepsilon+1)} exp\left\{\frac{-\beta_\varepsilon}{\sigma_\varepsilon^2}\right\} \times \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}(\sigma_1^2)^{-(\alpha_1+1)} exp\ \{\frac{-\beta_1}{\sigma_1^2}\}$$

$$\propto (\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} exp - \left\{\frac{\frac{1}{2}(Y - F\hat{\theta})^T Q(Y - F\hat{\theta}) + \beta_\varepsilon}{\sigma_\varepsilon^2}\right\} \times (\sigma_1^2)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)}$$

$$exp - \left\{\frac{\frac{1}{2}(Y - F\hat{\theta})^T P(Y - F\hat{\theta}) + \beta_1}{\sigma_1^2}\right\} exp\ \{-\frac{1}{2}(\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})\}.$$

From this expression, we can deduce the following conditional and marginal posterior distributions

$$\pi_1(\theta|\sigma_\varepsilon^2, \sigma_1^2, Y) \propto exp\ \{-\frac{1}{2}(\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})\}, \quad \textbf{(7)}$$

And

$$\pi_1(\sigma_\varepsilon^2|\theta,\sigma_1^2,Y) \propto (\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} exp\left\{-\frac{\frac{1}{2}(Y-F\hat{\theta})^T Q(Y-F\hat{\theta})+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}, \quad (8)$$

$$\pi_1(\sigma_1^2|\theta,\sigma_\varepsilon^2,Y) \propto (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} exp\left\{-\frac{\frac{1}{2}(Y-F\hat{\theta})^T P\,(Y-F\hat{\theta})+\beta_1}{\sigma_1^2}\right\} \quad (9)$$

Therefore, it follows that

$$\left(\theta\middle|\sigma_{\varepsilon,}^2\sigma_1^2,Y\right)\sim N\left(\hat{\theta},(F^T\Psi^{-1}F)^{-1}\right), \quad (10)$$

$$(\sigma_\varepsilon^2|\,\theta,\sigma_1^2,Y)\sim IG\left(\alpha_\varepsilon+\frac{N(T-1)}{2},\beta_\varepsilon+\frac{1}{2}\,(Y-F\hat{\theta})^T Q\,(Y-F\hat{\theta}\,)\right), \quad (11)$$

$$(\sigma_1^2|\theta,\sigma_\varepsilon^2,Y)\sim IG\left(\alpha_1+\frac{N}{2},\beta_1+\frac{1}{2}\,(Y-F\hat{\theta})^T P\,(Y-F\hat{\theta})\right) \quad (12)$$

Bayes Factor

We would like to choose between a fully Bayesian panel data model with $(K+1)$of parameters against a Bayesian panel data model with $(q+1)$ofparameters, where$q<K$, by using Bayes factor for two hypotheses

$$\text{against} \left.\begin{matrix} H_0: Y_{it}=\beta_0\ +\ \sum_{j=1}^{q}\beta_j x_{jit}+\omega_{it}, or\ H_0: F^0\theta^0+\omega \\ H_1: Y_{it}=\beta_0\ +\ \sum_{j=1}^{K}\beta_j x_{jit}+\omega_{it}, or\ H_1: F\theta+\omega \end{matrix}\right\}, \quad (13)$$

Where, θ^0 is $(q+1)$ vectors of parameters, F^0 is an $NT\times(q+1)$ design matrix and $q<K$. We compute the Bayes factor, B_{01} of H_0 relative to H_1 for testing problem (13) as follows

$$B_{01}=\frac{m\,(Y|H_0)}{m(Y|H_1)}, \quad (14)$$

Where $m\,(Y|H_i)$ is the marginal density of Yunder model $H_i, i=0,1$. From [10] we have:

$$m(Y|\,H_0)=\iint\left(\int f\,\left(Y|\theta^0,\sigma_\varepsilon^2,\sigma_1^2\right)\pi_1(\theta^0|\sigma_\varepsilon^2,\sigma_1^2)\pi_0(\sigma_\varepsilon^2,\sigma_1^2)d\theta^0\right)d\sigma_\varepsilon^2 d\sigma_1^2).$$

$$=(2\pi)^{\frac{-NT}{2}}\frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}\frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)}\Gamma\left(\frac{N(T-1)}{2}+\alpha_\varepsilon\ +2\right)\left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\ \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\Gamma\left(\frac{N}{2}+\alpha_1\ +2\right)\left(\frac{1}{2}\,(Y-F^0\theta^0)^T P(Y-F^0\theta^0)+\beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}.$$

$$m(Y|H_1)=\iint(\int f\,(Y,\theta,\sigma_\varepsilon^2,\sigma_1^2)\,\pi_1(\theta|\sigma_\varepsilon^2,\sigma_1^2)\pi_0(\sigma_\varepsilon^2,\sigma_1^2)d\theta)d\sigma_\varepsilon^2 d\sigma_1^2.$$

$$=(2\pi)^{\frac{-NT}{2}}\frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}\frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)}\Gamma\left(\frac{N(T-1)}{2}+\alpha_\varepsilon\ +2\right)\left[\frac{1}{2}\,(Y-F\theta)^T Q(Y-F\theta)+\ \beta_\varepsilon\right]^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\times\Gamma\left(\frac{N}{2}+\alpha_1\ +2\right)\left(\frac{1}{2}\,(Y-F\theta)^T P(Y-F\theta)+\ \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}.$$

$$\therefore B_{01}=\frac{\left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\ \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}\left(\frac{1}{2}\,(Y-F^0\theta^0)^T P(Y-F^0\theta^0)+\ \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}}{\left(\frac{1}{2}\,(Y-F\theta)^T Q(Y-F\theta)+\ \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}\left(\frac{1}{2}\,(Y-F\theta)^T P(Y-F\theta)+\ \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}}$$

The Data Results

To illustrated the effectiveness of the methodology. We have chosen a data set from gross fixed capital formation and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.).The data are analysed according to our methodology by using gretl, R and matlab softwares. We used gross fixed capital formation at current prices for the years (2005-2015) (Million I.D.) for all economic activities as a dependent variables, gross fixed capital formation at current prices for the years (2005-2015) (Million I.D.) for previous year and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.) as explanatory variables, as well as, we consider the economic activities as sections. Then we have (9) sections, where

Section (1): agriculture, forestry, hunting and fishing,

Section (2): mining and quarrying,

Section (3): manufacturing industry,

Section (4): electricity and water,

Section (5): building and construction,

Section (6): transport communications and storage,

Section (7): wholesale, retail trade, hotels and others,

Section (8): banks and insurance,

Section (9): social and personal services.

This application has been divided into two parts. The first part includes the estimation of sub-models for sections. By using gretl software we obtain the maximum likelihood estimators for the sub-models of the sections both individually and the total model for all sections together. Table (1) below shows the results for the model estimators. From this table we can see that the models for building and construction and wholesale, retail trade, hotels and others where significance at a 0.05 level of significance. This means there is a significant effect for gross fixed capital formation at current prices for the years (2005-2015) (Million I.D.) for previous year and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.) on the gross fixed capital formation at current prices for the years (2005-2015) (Million I.D.) for all economic activities. Furthermore, for the sections(1), (2),(3), (4),(6),(8),(9) which was non-significant, in fact this don't agree with the economic theory, then we can treat this problem by using confounding approach for this sections data (i.e. the data of the economic activities: agriculture, forestry, hunting and fishing, mining and quarrying, manufacturing industry, electricity and water, transport, communication and storage, banks and insurance, social and personal services) with time series to obtain (29) observation. Table (2) shows the results the total model estimator for all the economic activities, clearly from this table the model is significance at a 0.05 level of significance according to F- value which (20.27009) with p- value (F) which (7.00e-19), as well as the value of determinant coefficient ($R^2 = 0.70$), this means the total model is agree with the economic theory. Also table (3) presents summary values estimation of the confounded model. From this table we can see that the total model after confounding also significance at a 0.05 level of significance. The second part we applied our methodology (Bayesian method) to the data set from gross fixed capital formation and gross domestic product by economic activities for public sector at current prices for the years

(2005-2015) (Million I.D.). Figure(1) represents the posterior density of the coefficients for the model of section(1) (building and construction) and figure (2) shows the number of iterations of the Gibbs sampler used in this study, which was (7000) iterations, while figure (3) shows density estimates based on (7000) iterations of σ_ε^2,σ_1^2 and σ_u^2 for this model. We have displayed posterior density of coefficients for the model of section (1) (wholesale, retail trade, hotel and others) in figure(4) and figure (5) shows of iterations of the Gibbs sampler of this model which was (7000). Figure (6) represents the posterior density based on (7000) iterations of σ_ε^2,σ_1^2 and σ_u^2 for the model of section (2). Furthermore, the figures (7) and (10) represents the posterior density of the coefficients for the total model and the confounded model respectively, also the figures (8) and (11) shows the number for the iterations of the Gibbs sampler which was (5000) and (8000) for this models respectively and figures (9) and (12) shows the posterior density based on (5000)and(8000) iterations of σ_ε^2,σ_1^2 and σ_u^2 for the total model and the confounded model respectively. Table (4) presents the values of the parameters for the models estimates based on the Bayesian method. From this table, we can see that the values of parameters obtained by Bayesian method are encouraging.

Table 1: The Model Estimators for Sections (Economic Activities)

Section	Parameter estimator	The value of estimators	Standard error	t- test	p-value	F − test	p −value (F)	R^2
1	$\hat{\beta}_0$	3.86660e+07	2.55386e+08	0.1514	0.8834	0.213435	0.812260	0.050656
	$\hat{\beta}_1$	0.0675264	0.335536	0.2012	0.8455			
	$\hat{\beta}_2$	17.0406	28.9661	0.5883	0.5726			
2	$\hat{\beta}_0$	-2.14066e+08	5.75331e+08	0.3721	0.7195	0.785175	0.488257	0.164085
	$\hat{\beta}_1$	0.0835356	0.163748	0.5101	0.6237			
	$\hat{\beta}_2$	12.2181	9.76379	1.251	0.2462			
3	$\hat{\beta}_0$	1.10795e+010	6.58628e+09	1.682	0.1310	0.880369	0.451263	0.180390
	$\hat{\beta}_1$	-2.45741	3.28918	-0.7471	0.4764			
	$\hat{\beta}_2$	- 1324.87	1456.62	-0.9095	0.3896			
4	$\hat{\beta}_0$	1.28578e+09	1.52223e+09	0.8447	0.4228	4.093918	0.059649	0.505802
	$\hat{\beta}_1$	0.606713	0.237005	2.560	0.0337			
	$\hat{\beta}_2$	170.653	137.960	1.237	0.2512			
5	$\hat{\beta}_0$	-2.00648e+08	1.60905e+08	-1.247	0.2477	7.638054	0.013955	0.656300
	$\hat{\beta}_1$	-0.494557	0.301930	-1.638	0.1401			
	$\hat{\beta}_2$	64.7116	17.6098	3.675	0.0063			
6	$\hat{\beta}_0$	6.51350e+08	1.64660e+09	0.3956	0.7028	1.215018	0.346112	0.232984
	$\hat{\beta}_1$	0.386163	0.504278	0.7658	0.4658			
	$\hat{\beta}_2$	37.8384	181.875	0.2080	0.8404			
7	$\hat{\beta}_0$	2.02903e+08	7.13551e+07	2.844	0.0217	12.04065	0.003867	0.750634
	$\hat{\beta}_1$	1.18771	0.261094	4.549	0.0019			
	$\hat{\beta}_2$	-18.9722	7.96265	2.383	0.0444			
8	$\hat{\beta}_0$	9.36255e+07	2.92096e+07	3.205	0.0125	2.066738	0.188981	0.340667
	$\hat{\beta}_1$	-0.790978	0.404201	-1.957	0.0861			
	$\hat{\beta}_2$	12.5969	9.70421	1.298	0.2304			
9	$\hat{\beta}_0$	6.70564e+09	5.63077e+09	1.191	0.2678	1.239066	0.339801	0.236505
	$\hat{\beta}_1$	0.420409	0.301515	1.394	0.2007			
	$\hat{\beta}_2$	52.6157	197.660	0.2662	0.7968			

Table 2: The Total Model Estimator

Parameter Estimator	The Value of Estimators	Standard Deviation	$t-$Test	p $-$Value	F $-$Test	$p-$Value(F)	R^2
$\hat{\beta}_0$	8.26169e+08	5.45353e+08	1.515	0.1334	20.27009	7.00e-19	0.697283
$\hat{\beta}_1$	0.390519	0.0933341	4.184	6.76e-05			
$\hat{\beta}_2$	48.4406	26.5657	1.823	0.0716			

Table 3: The Total Model Estimator After Confounded

Parameter Estimator	The Value of Estimators	Standard Deviation	$t-$Test	p $-$Value	F $-$Test	$p-$Value(F)	R^2
$\hat{\beta}_0$	-1.21363e+08	9.27811e+07	-1.308	0.2023	9505.042	5.76e-38	0.998634
$\hat{\beta}_1$	1.04514	0.00878075	119.0	4.06e-037			
$\hat{\beta}_2$	6.17952	0.540683	11.43	1.23e-011			

Table 4: Estimation Values by Bayesian Method for the Models

The Model	The Parameter	The Value of Parameter
Model of section1	$\hat{\beta}_0$	-2.00626e+08
	$\hat{\beta}_1$	-0.494
	$\hat{\beta}_2$	64.71
Model of section2	$\hat{\beta}_0$	2.02091e+08
	$\hat{\beta}_1$	1.19
	$\hat{\beta}_2$	-18.97
Total model	$\hat{\beta}_0$	8262e+08
	$\hat{\beta}_1$	0.3905
	$\hat{\beta}_2$	48.44
Confounded model	$\hat{\beta}_0$	-1214e+08
	$\hat{\beta}_1$	1.045
	$\hat{\beta}_2$	6.18

The model checking approach based on Bayes factors has been tested on estimated models. These Bayes factors are given in table (5). From this table, it can be seen that the Bayes factors favors H_1 with strong evidence with all models for the data of gross fixed capital formation and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.).

Table 5: Shows Bayes Factor For H_0: $\mathrm{F}^0\theta^0+\omega$ $varsus$ H_1: $\mathrm{F}\theta+\omega$

Models	B_{01}	Evidence
Model of section1	$1.875279497000141\times10^{-25}$	very strongly favors H_1
Model of section2	$7.903255738497785\times10^{-9}$	very strongly favors H_1
Total model	$8.946564506647714\times10^{-34}$	very strongly favors H_1
Confounded model	$6.115669635463685\times10^{-32}$	very strongly favors H_1

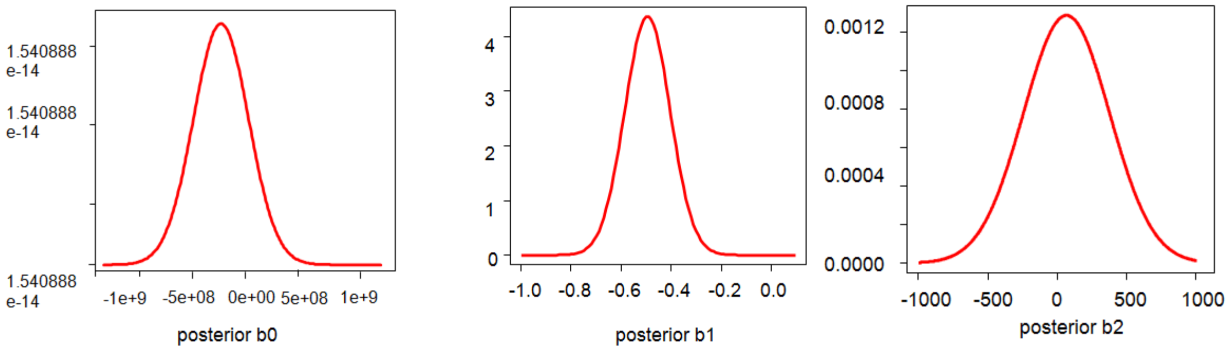

Figure 1: Posterior Density of the Coefficients for the Model of Section1

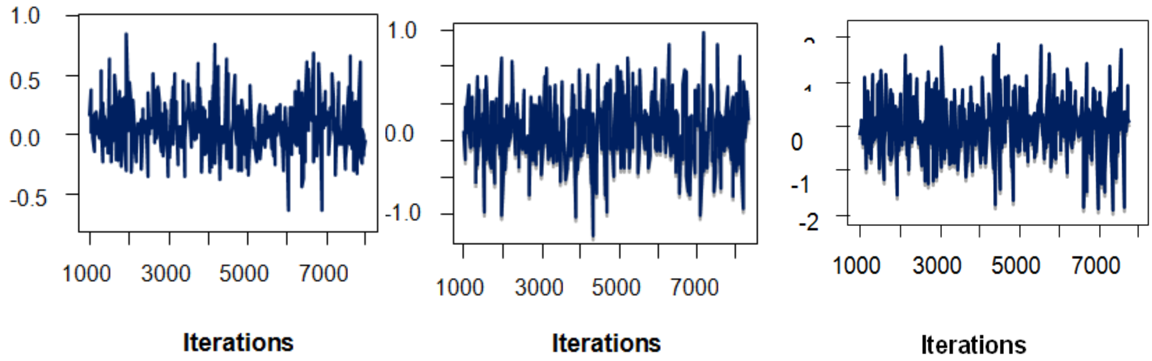

Figure 2: Shows (7000) Iteration of the Gibbs Sampler for the Model of Section1

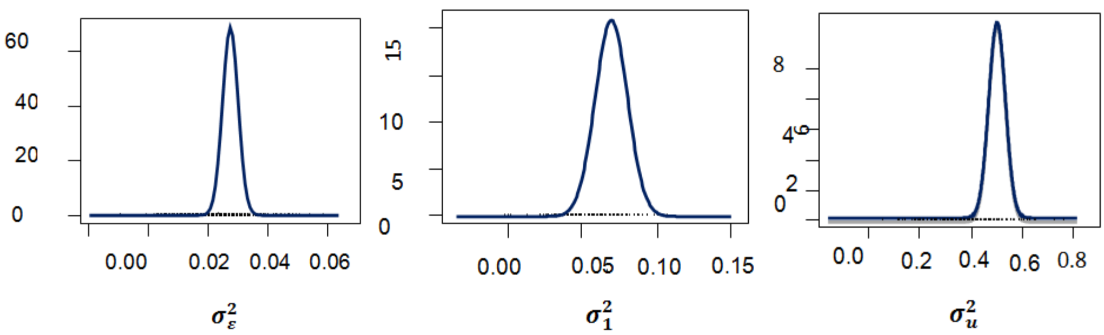

Figure 3: Shows the Density Based on (7000) Iteration of σ_ε^2,σ_1^2 and σ_u^2 the Gibbs Sampler for the Model of Section1

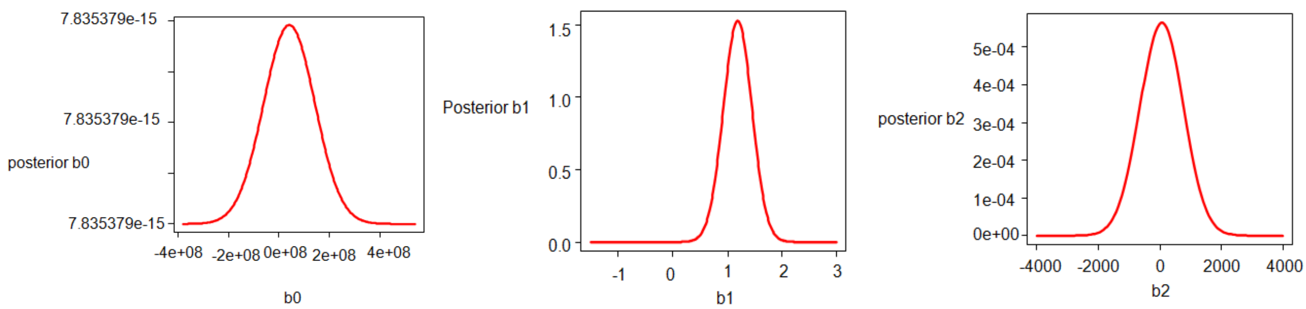

Figure 4: Posterior Density of the Coefficients for the Model of Section2

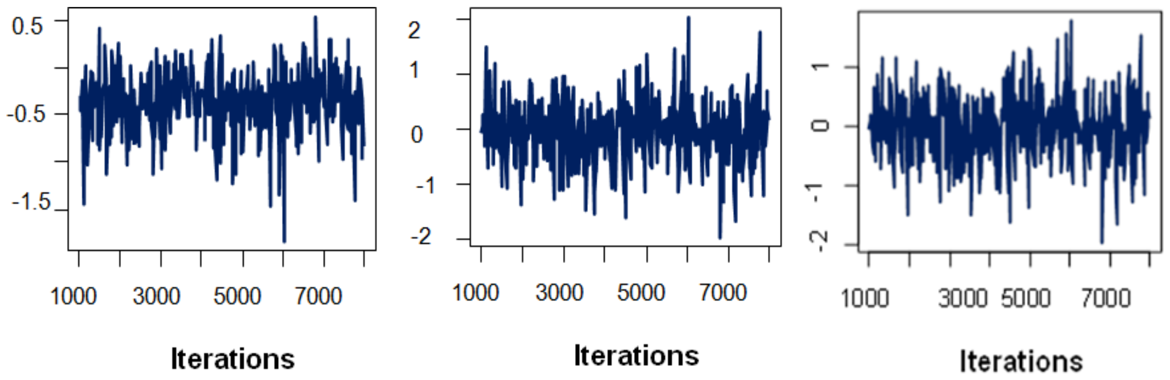

Figure 5: Shows (7000) Iteration of the Gibbs Sampler for the Model of Section2

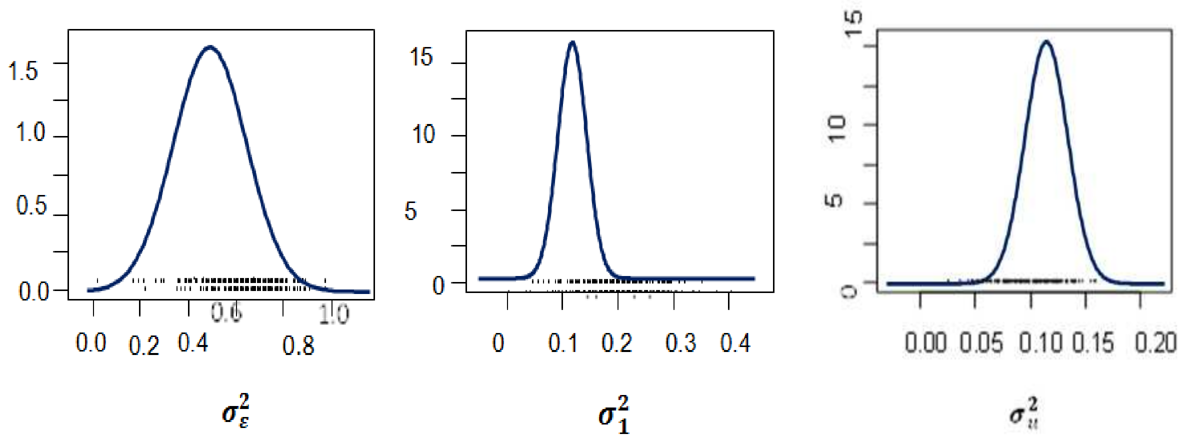

Figure 6: Shows the Density Based on (7000) Iteration of σ_ε^2,σ_1^2 and σ_u^2 the Gibbs Sampler for the Model of Section2

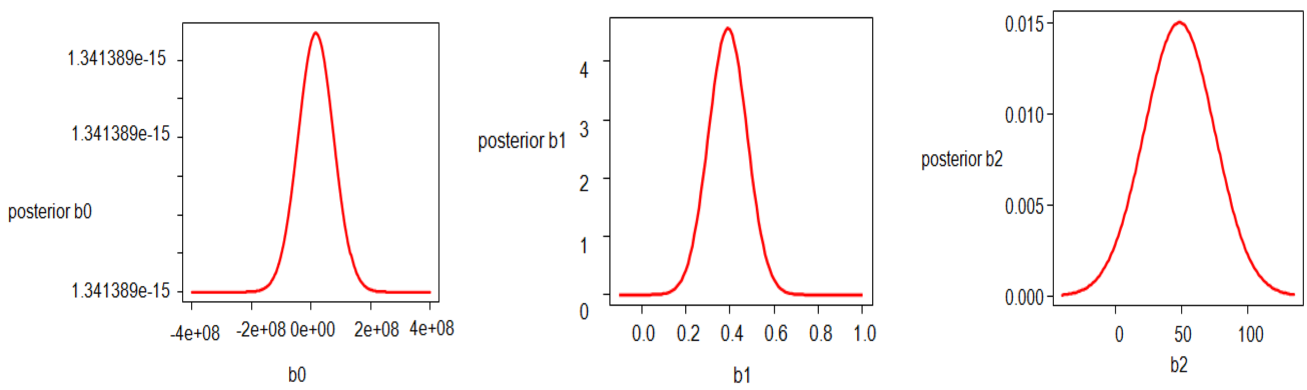

Figure 7: Posterior Density of the Coefficients for total Model

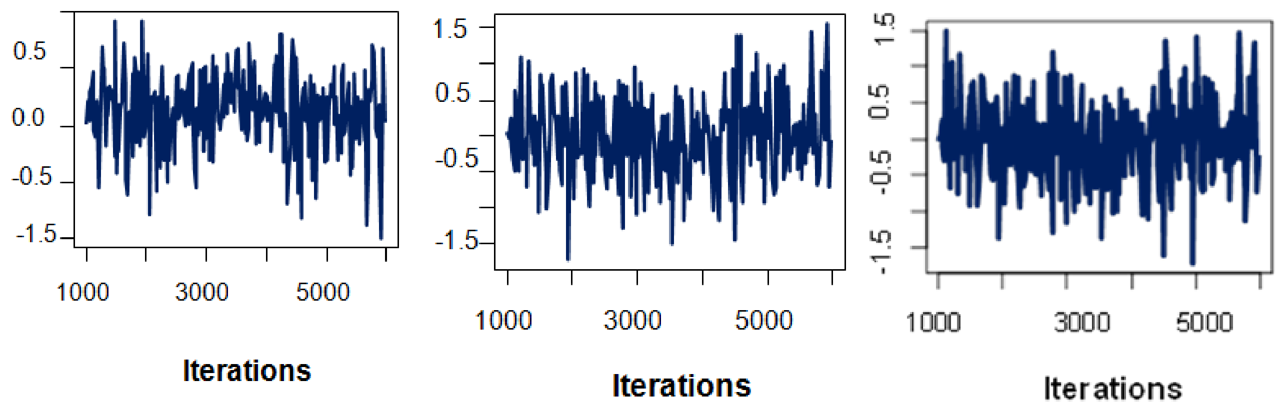

Figure 8: Show (5000) Iteration of the Gibbs Sampler for the Total Model

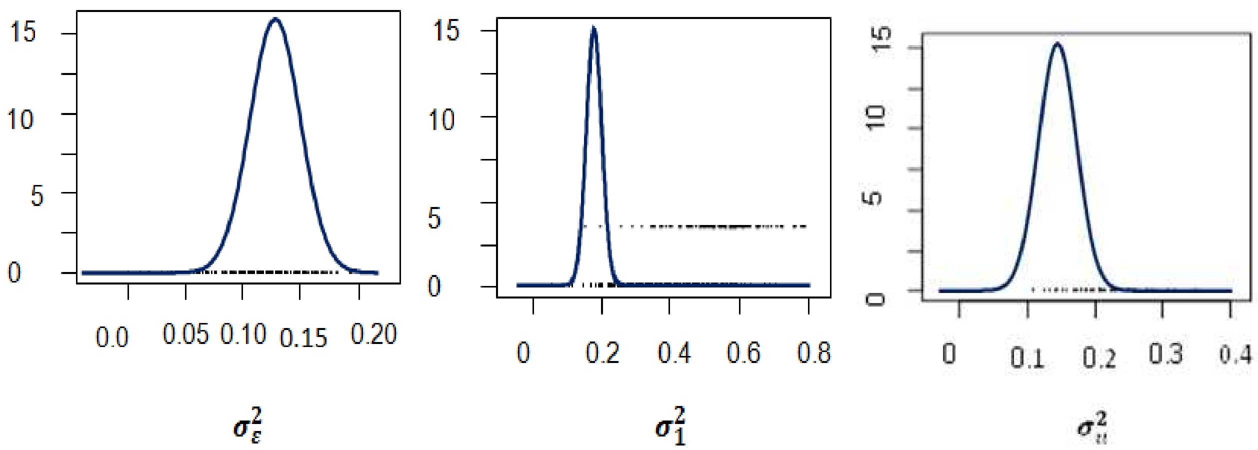

Figure 9: Shows the Density Based on (5000) Iteration of σ_ε^2,σ_1^2 And σ_u^2 the Gibbs Sampler

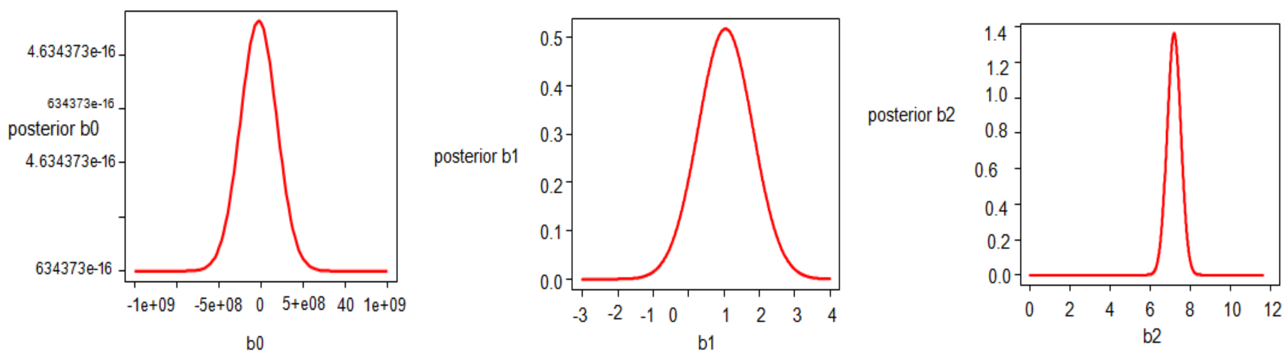

Figure 10: Posterior Density of the Coefficients for Confounded Model

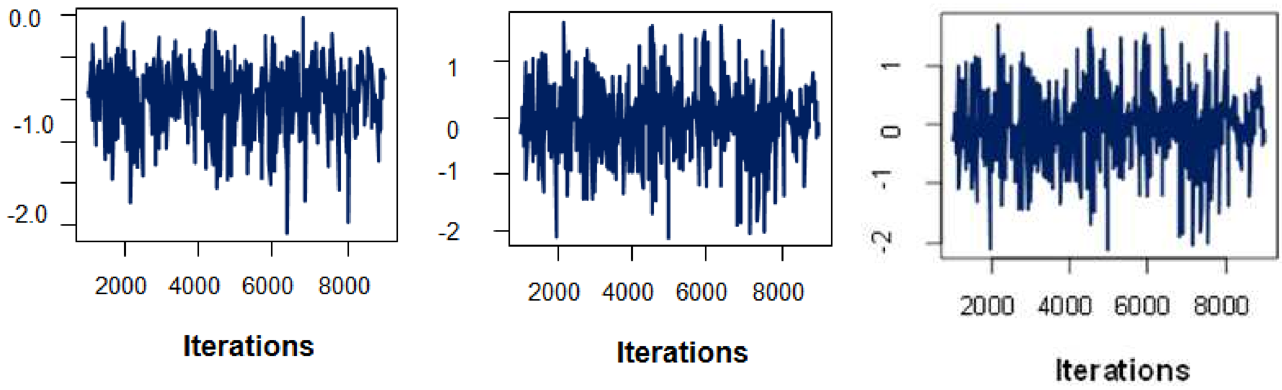

Figure 11: Show (8000) Iteration of the Gibbs Sampler for the Confounded Model

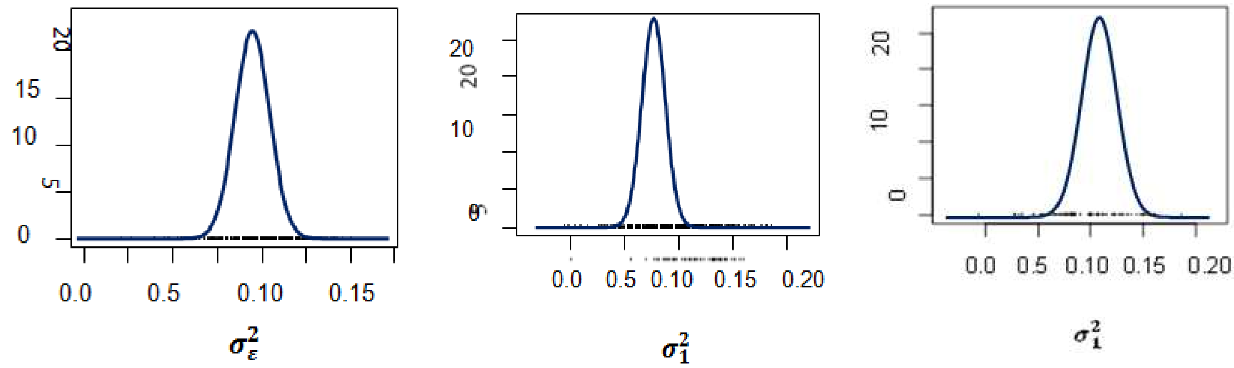

Figure 12: Shows the Density Based on (8000) Iteration of σ_ε^2,σ_1^2 and σ_u^2 the Gibbs Sampler

CONCLUSIONS

The conclusions which are obtained throughout this paper are given as follows:

- The models for building and construction and wholesale, retail trade, hotels and others where significance at a 0.05 level of significance. This means there is a significant effect for gross fixed capital formation at current prices for the years (2005-2015) (Million I.D.) for previous year and gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.) on the gross fixed capital formation at current prices for the years (2005-2015) (Million I.D.) for all economic activities
- The total model estimator for all the economic activities, is significance at a 0.05 level of significance according to F- value which (20.27009) with p- value (F) which (7.00e-19), as well as the value of determinant coefficient ($R^2 = 0.70$), this means the total model is agree with the economic theory.
- The total model after confounding also significant at a 0.05 level of significance.
- The values of the parameters for the models estimates based on the Bayesian method, were encouraging.
- The Bayes factors favors H_1 with strong evidence with all models for the data of gross fixed capital formation and

gross domestic product by economic activities for public sector at current prices for the years (2005-2015) (Million I.D.).

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